Convergence of the ruin and related quantities of interest in Lévy insurance risk analysis

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Summary: This paper is concerned with asymptotic behavior of limiting distributions of a wide range of quantities of interest in insurance risk analysis, the claim causing ruin etc. Our aim is to estimate, and derive, where necessary, limiting distributions of these quantities with respect to Lévy insurance risk model. We also discuss the large-time behavior (passage above a high level) of the ruin time, and related quantities, viz, the jumps when cause the ruin. Most results will assume "convolution equivalent" (related to subexponential) behavior of the tail of the Lévy measure. Rates of convergence to asymptotic distributions are considered by simulation of finite sample versions. In this analysis, throughout, we follow the notation of Doney and Kyprianou [Ann. Appl. Probab. 2006] and Park and Maller [Adv. Appl. Probab. 2008] for the most part.
By the quintuple law of Doney and Kyprianou (2006) we have the following results for $u > 0$:

(a)
$$P\left(X_{\tau(u)} - \overline{X}_{\tau(u)-} < x; \tau(u) < \infty\right) = \int_{y\in(0,x]}\int_{v\in(y,\infty)} \left(\Pi_X(x-y+v) - \Pi_X(v)\right)\widehat{V}(\mathrm{d}v - y)V(u-\mathrm{d}y). \tag{1}$$

(b)
$$\lim_{u\to\infty} P^{(u)}\left(X_{\tau(u)} - \overline{X}_{\tau(u)-} < x\right) = \frac{\mathrm{d}_{\mathcal{H}}\alpha}{q} - \widetilde{g}_\alpha(x) + \frac{1}{q}\int_{y\in(0,x]}\left(e^{\alpha y} - 1\right)\Pi_{\mathcal{H}}(\mathrm{d}y), \tag{2}$$

where $\widetilde{g}_\alpha(x) = \overline{\Pi}_{\mathcal{H}}(x)\left(\frac{1}{\alpha}e^{\alpha x} - 1\right)$, and $\mathrm{d}_{\mathcal{H}}$ is the drift of the nondefective ladder height process, $\mathcal{H}$. Let $P^{(u)}$ denote probability and expectation, conditional $\tau(u) < \infty$, where $\tau(u)$ is the first passage time above level u: $\tau(u) = \inf\{t \geq 0 : X_t > u\}$, for $u > 0$. Here the amount of jumps is of the quantities $X_{\tau(u)} - X_{\tau(u)-}$ and $X_{\tau(u)} - \overline{X}_{\tau(u)-}$ conditional on $\tau(u) < \infty$, where $\overline{X}_t = \sup_{0<s\leq t} X_s$. Let τ_q be an exponentially distributed random time with parameter $q > 0$, which is independent of the Lévy process. We then obtain for $\alpha > 0$ and $x \geq 0$,

$$P\left(\tau(u) > t\right) = \frac{1}{q}\kappa(q,0)\int_{(0,u]} P(\mathcal{H}_t \in \mathrm{d}y), \tag{3}$$

where we denote by $\kappa(p,r)$ joint Laplace exponents.
Keywords: Lévy insurance risk process; Ladder height process; Bivariate subordinator; Subexponential and Convolution equivalent distributions; Quintuple law; Renewal measure.

References

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